

Stock Assessment of US Atlantic Wreckfish

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Introduction

This stock assessment is being conducted as part of a Climate Response Strategy for a Data-Limited Fishery. The last wreckfish stock assessment was based on data up to 2010 (Rademeyer & Butterworth 2014). The primary assessment tool was a statistical age/length model, with a dynamic production model used as a secondary tool. The current assessment will provide an update with 13 years of additional data.

Data sources

Landings

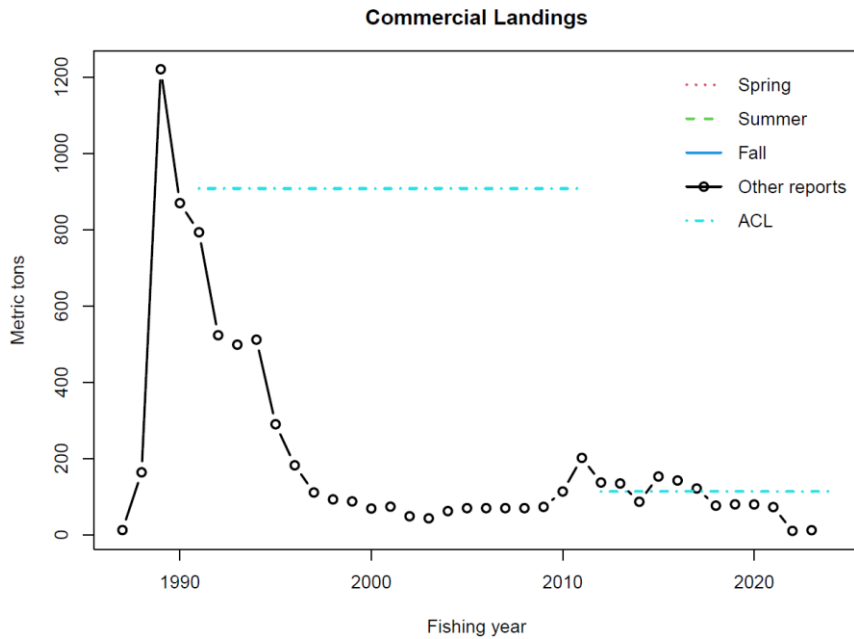


Figure 1. Commercial wreckfish landings reported by Rademeyer & Butterworth (2014) and by SAMFC (2019). Points from 2000 to 2008 are the average for that period due to low number of participating vessels. Updated landings data from vessel trip reports are not shown here due to confidentiality concerns in some years. Landings have been converted from pounds to metric tons in preparation for the stock assessment model.

Most of the data used in the assessment come from the SAFIS database. These data files are up to date and have been validated by agency scientists. Landings data come from commercial trip reports and dealer reports, as no data are available on recreational catches or discards. These two components are assumed to be minor. Commercial landings from 1987 to 1990 were taken from Rademeyer & Butterworth (2014) and

SAMFC (2019). The wreckfish fishing year runs from 15 April to 15 January the following year. Landings are consistently reported in the first two weeks of January, presumably because fishermen use coupons that would otherwise expire. Data are therefore reported by fishing year, rather than calendar year. The commercial landings data obtained from vessel trip reports are consistent with previously reported wreckfish landings (Fig. 1). Landings data from the most recent decade range between 45 and 160 metric tons.

Wreckfish are not regularly caught by any of the NOAA Fisheries surveys, such that a survey abundance index is not available. Vessel trip reports contain measures of fishing effort that can be used to construct a landings-per-unit-effort (LPUE) index as well as covariates that may influence fishing success. Prior to calculating a LPUE index, we examined the spatio-temporal patterns of potential explanatory variables (Fig. 2).

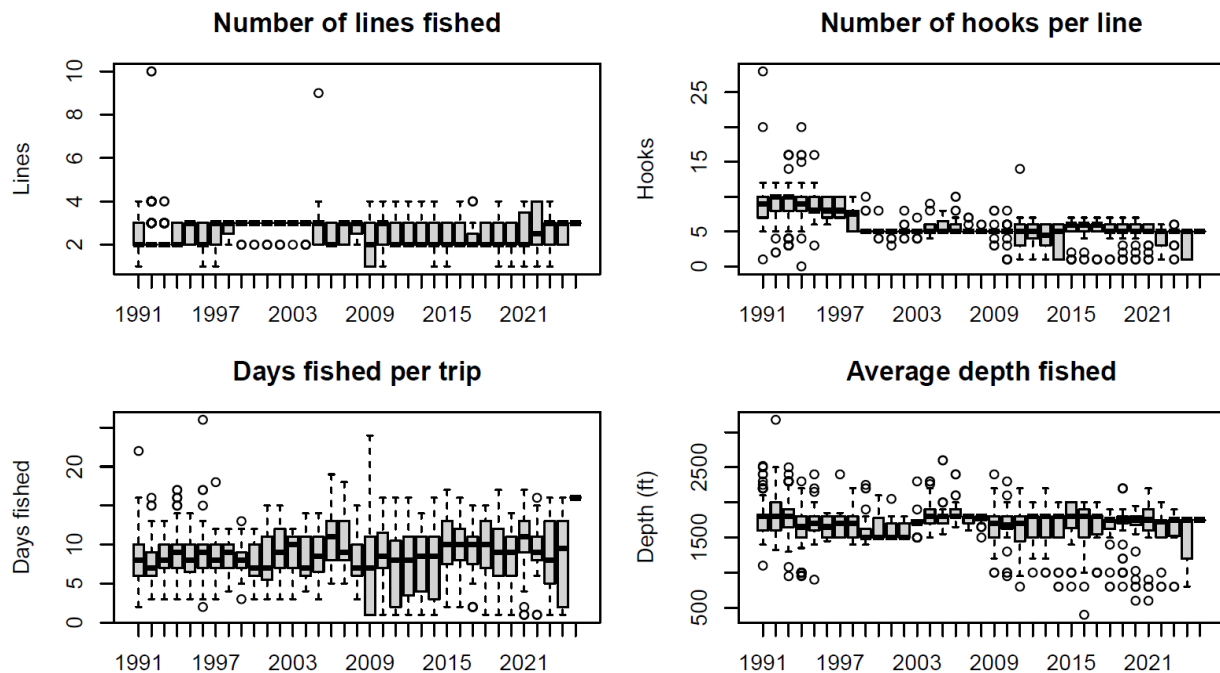


Figure 2. Potential covariates of fishing success. Box plots show the median (black line) interquartile ranges (boxes), range (dashed lines), and outliers (points).

With few exceptions, the median number of lines fished has ranged between 2 and 3. The number of hooks per line was higher in the early years before dropping to a median of five. The median days per trip has varied between 5 and 10 days, with more variation since 2009. Depth fished is reported as an average for a trip, which helps to explain the low variation between 1500 to 1800 ft. Some deeper depths were reported in the earlier years and shallower depths in the recent years.

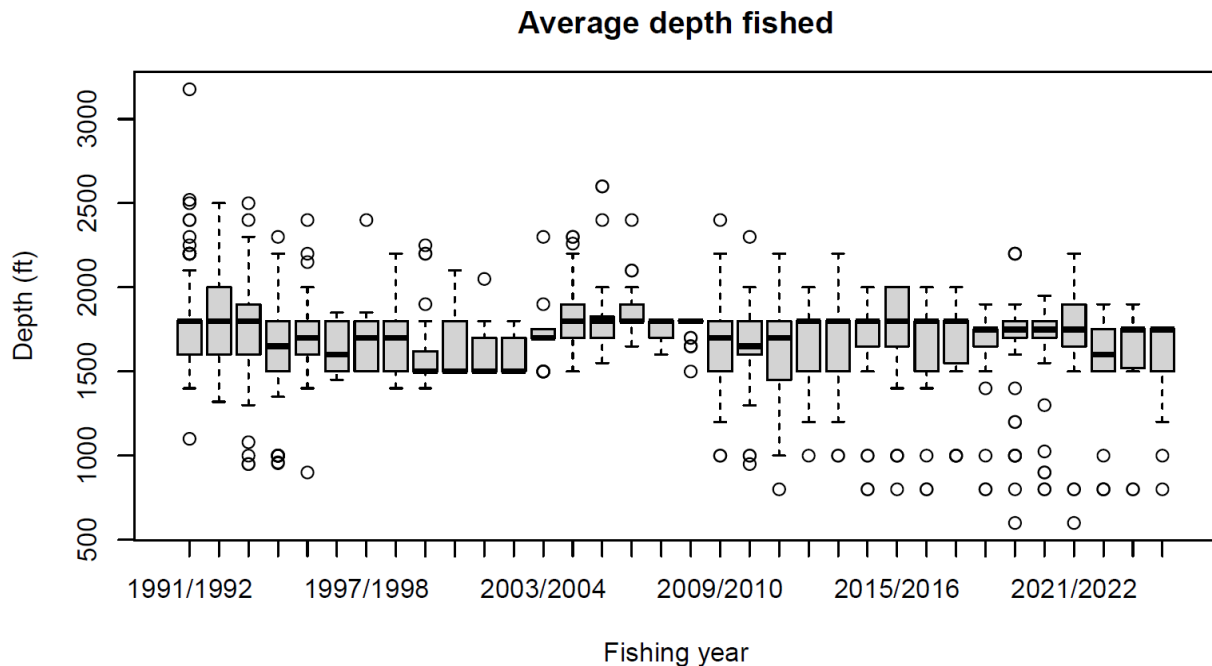


Figure 3. Average depth fished per fishing trip. Box plots show the median (black line) interquartile ranges (boxes), range (dashed lines), and outliers (points).

Looking more carefully at the depth fished (Fig. 3), there was discussion of some of the outliers at the November 2005 AIM workshop. Regarding the depth > 3000 ft reported in 1991/1992, there was no area reported for this trip that could be used to verify the location. Fishermen felt that depths <1000 ft do not represent wreckfish habitat and suggested that additional species may have been targeted on these trips. The shallow depths were reported in areas off the Florida coast. One report of 400 ft was omitted from Fig. 3; based on other depths reported by that vessel in that area, it was more likely to be 800 ft.

Fishing area is reported with a resolution of 1° by 1°. Each area is designated with four digits: the first two are latitude and the second two longitude. A number of fishing area records were obviously misrecorded (e.g. too far north or south, on land, in Cuba). Of 2200 records that reported fishing area, there were four records with incorrect latitude and 11 with incorrect longitude. These records were corrected by cross-referencing the area(s) where a given vessel usually fished. All corrections involved a single digit or transposition of two digits (e.g. 2840 changed to 2480).

The map of fishing locations confirms that the wreckfish fishing is concentrated off the coast of South Carolina and Florida (Fig. 4).

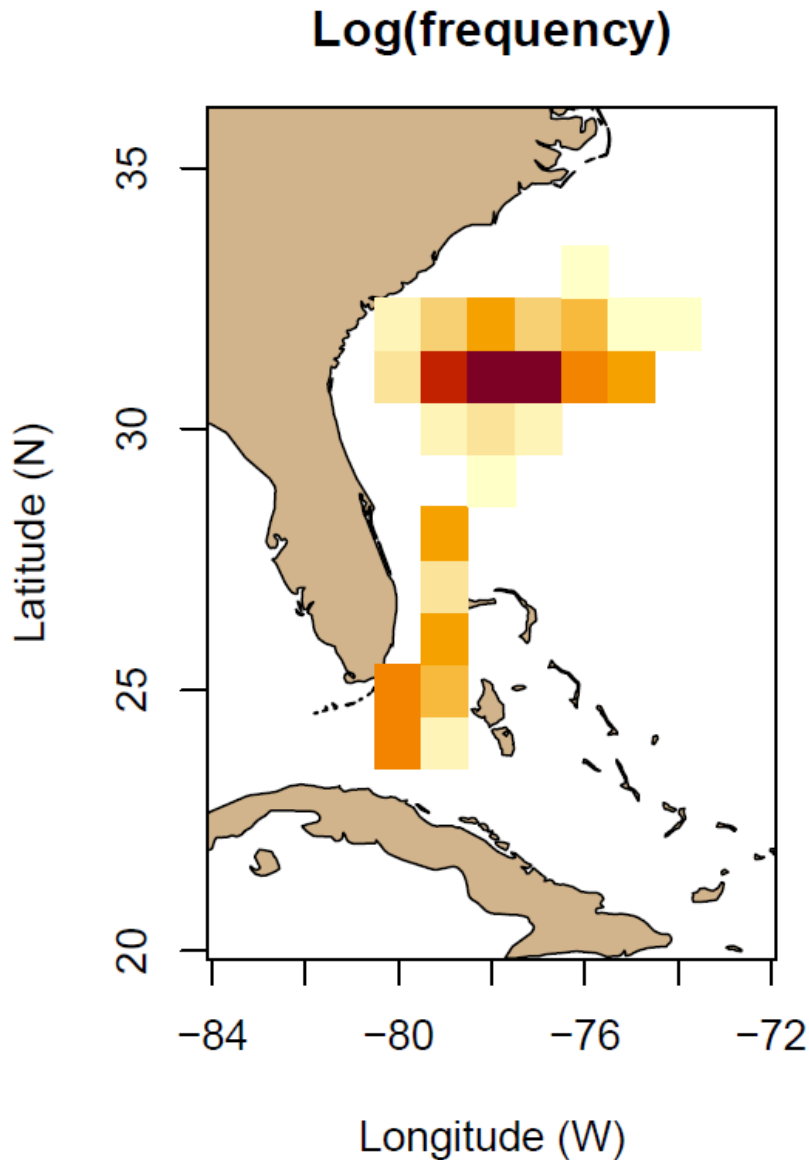


Figure 4. Fishing locations reported in vessel trip reports. Frequency of occurrence of each 1° by 1° area on a log scale. The color scheme runs from red (high) to yellow (low).

To examine how fishing locations may have changed over time, the frequency of fishing in each area was plotted against fishing year (Fig. 5). The Blake Plateau off the South Carolina coast has been consistently fished over time (Fig. 5). Areas to the north and south were fished in the early years and again in recent years. Areas off the Florida coast have been consistently fished since 2009.

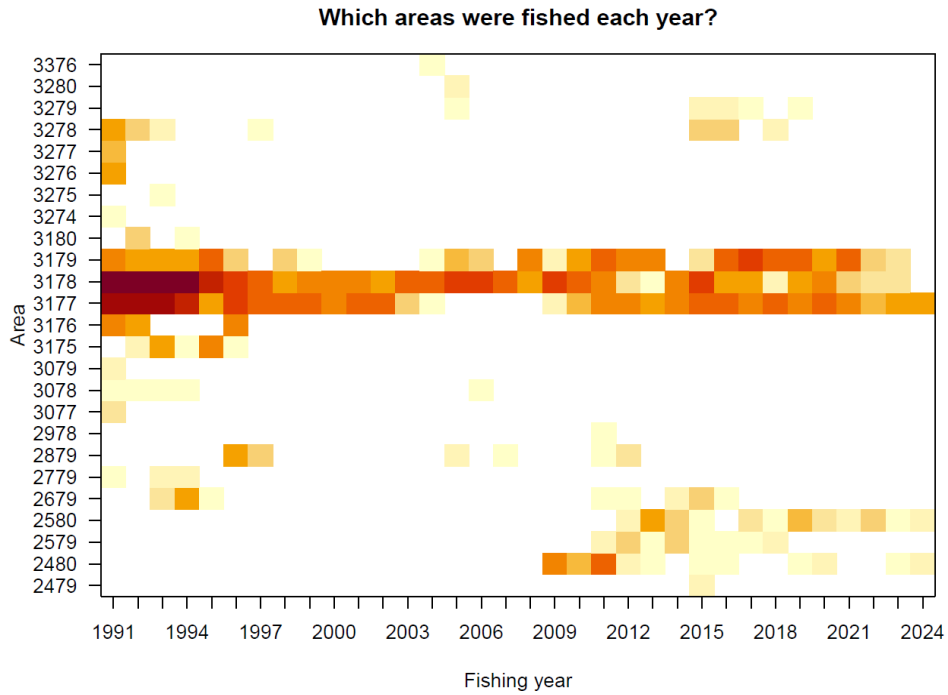


Figure 5. Fishing locations reported in vessel trip reports. Frequency of occurrence of each 1° by 1° area on a log scale. Areas are ordered from north (top) to south (bottom).

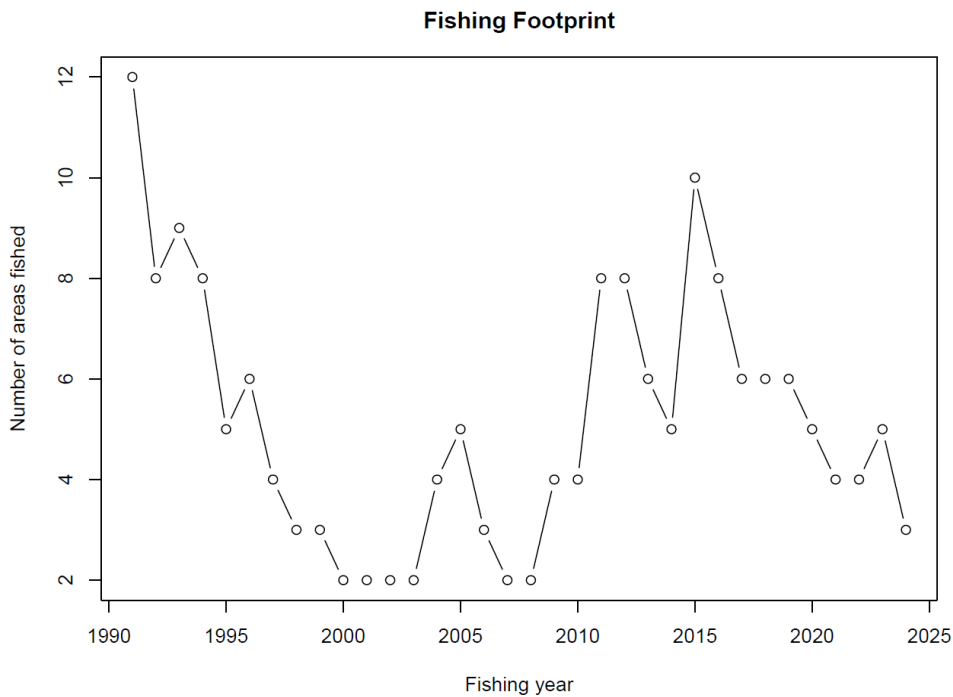


Figure 6. Spatial footprint of the wreckfish fishery estimated as the number of 1° by 1° areas fished each year.

The spatial footprint of the fishery was approximated by counting the number of 1-degree quadrants fished each year (Fig. 6). The spatial footprint ranged from a high of 12 in 1991 (representing 367,200 square miles) to a low of 2 areas in 2000-2003 and 2007-2008 (representing 7,200 miles). The fishing footprint increased from 2010 to 2015 before declining until 2024. These changes in the spatial footprint reflect changing vessel participation and the areas targeted by these vessels.

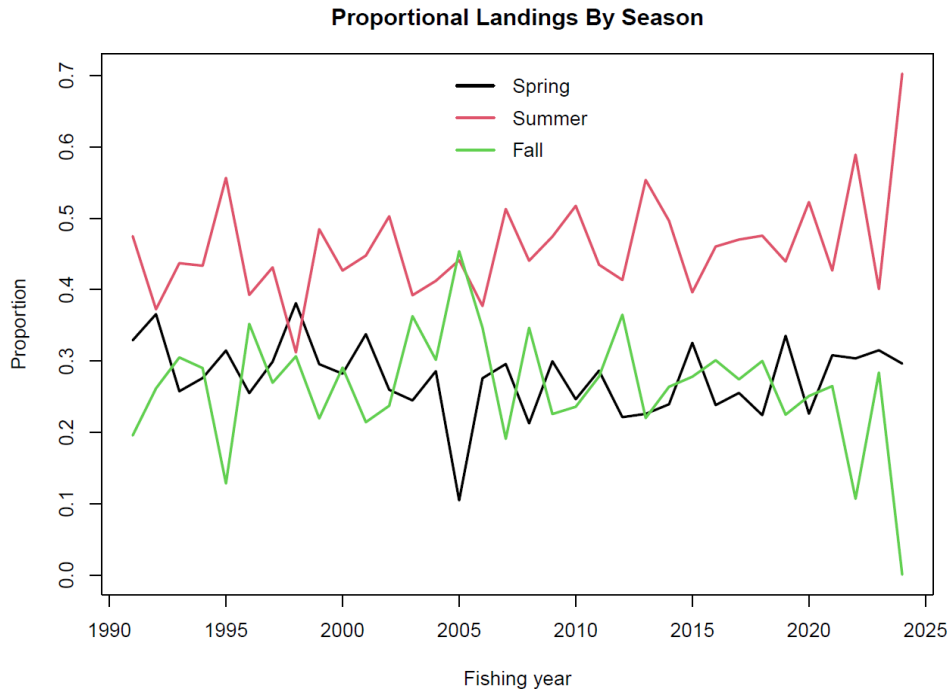


Figure 7. Proportional wreckfish landings by season. Spring = {April, May, June, July}, Summer = {August, September, October}, Fall = {November, December, January}.

The highest proportion of wreckfish landings (40-50%) are caught in the Summer months (Fig. 7). The proportional of wreckfish landings had varied among seasons but without a clear secular trend. In the most recent years, there was more catch in Summer and less in Fall.

The number of fishing vessels participating in the wreckfish fishery has varied from a high of 38 in 1991 to a low of 2 in 2001 and 2003 (Fig. 8). Since 2009 the number of participating vessels has varied between 3 and 7, depending on the number entering and exiting each year.

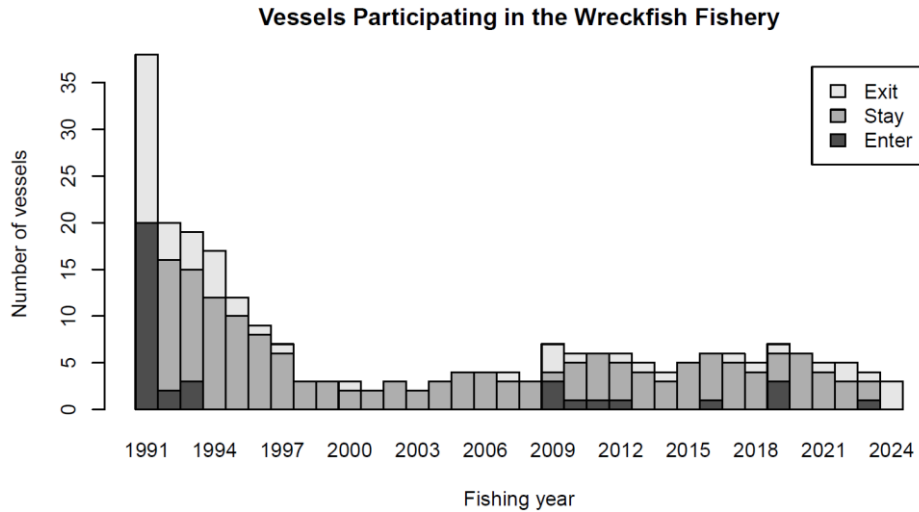


Figure 8. Participation in the wreckfish fishery. Bars at the start and end of the series need special interpretation. Some of the “entrants” in 1991 likely participated in 1990. Data for 2024 don’t anticipate vessels that will remain in the fishery.

Fishery-dependent abundance index

Catch per day (CPD) was selected as the response variable for developing a fishery-dependent abundance index. Days fished have been consistently reported in vessel trip reports with only one missing record. One high outlier of CPD was removed. This trip reported a catch of 11877 lb for one day fished. The catch amount is consistent with other trips, but the day fished seems like an error. The use of CPD as the response variable is consistent with previous LPUE indices used for wreckfish stock assessment (Vaughan et al. 2021, MacCall 2011).

Catch per day varied substantially among fishing areas and among fishing vessels (Fig. 9). Note that only single trips have been reported for a few areas and a few vessels. There is likely to be an area-vessel interaction, given that vessels fish in particular areas. Only one of these two variables is likely to be included in the final LPUE model. There is also a seasonal pattern to CPD, with catch rates highest in Fall and lowest in Summer. This seasonality is also apparent CPD by month, noting that February and March are completely closed to fishing. Only one measure of seasonality (season, month, day of year) will be included in the LPUE model.

Accounting for vessel effects in CPD is complicated by the turnover in vessel participation (Fig. 8). A straight average across vessels doesn’t account for differences in fishing success among vessels and their captains. For example, if the vessels with lower fishing success dropped out of the fishery, CPD of the remaining vessels would stay high even if abundance were decreasing (Fig. 10).

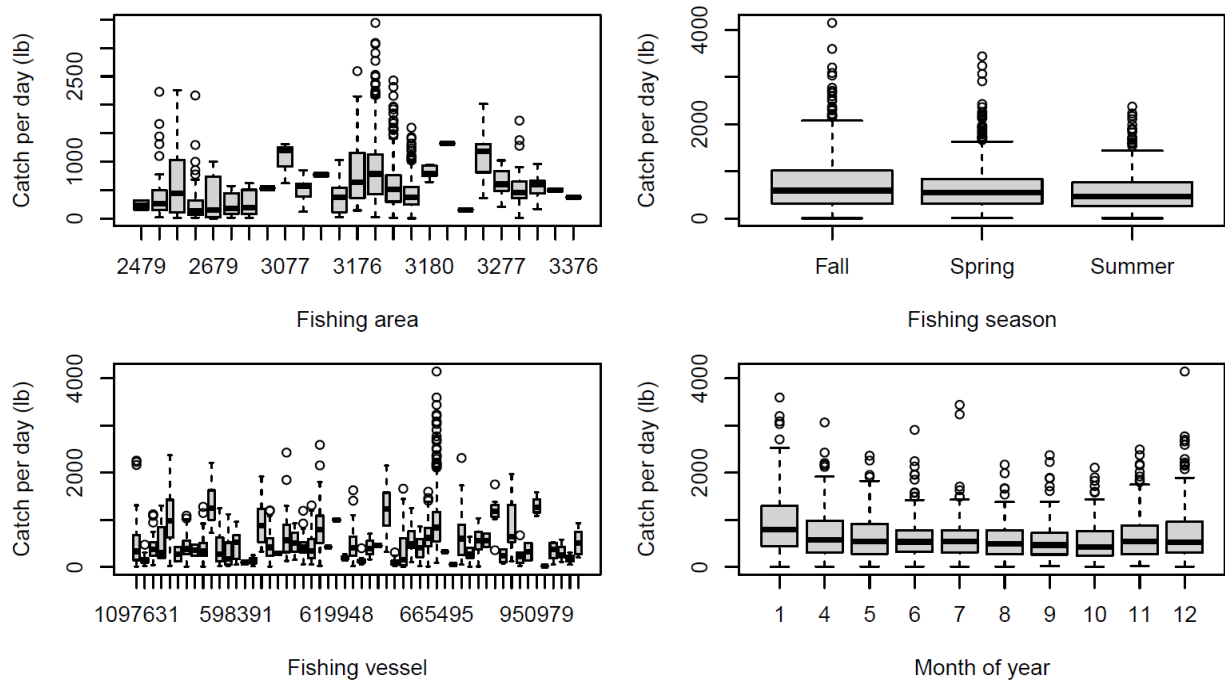


Figure 9. Catch per day (pounds) in relation to four potential covariates.

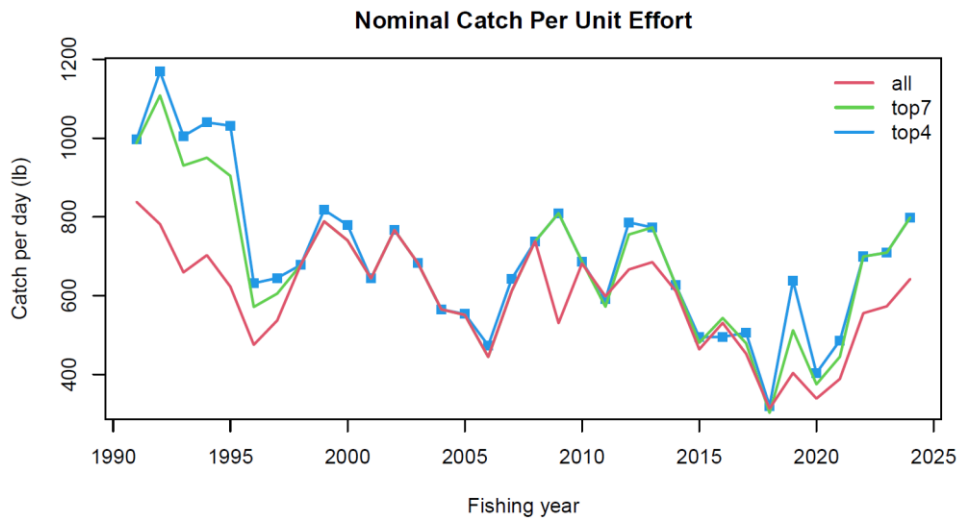


Figure 10. Catch per day (pounds) averaged by fishing year.

In calculating a LPUE index, it is desirable to use a subset of fishing vessels to decrease the correlation between LPUE and total catch. A subset of fishing vessels is chosen with catch rates that are thought to best represent the underlying changes in abundance. For wreckfish we considered the subset of vessels with the longest participation in the fishery. The top 4 vessels participated in the wreckfish fishery between 15 and 34 years. The top 7 added three vessels with 9 years of participation each. The yearly pattern of CPD is very similar for the top4 and top7 vessels (Fig. 10).

Adding three vessels to the top 4 would not add much information but would require three more vessel coefficients to be estimated. By contrast, the top-4 vessels have more overlap in the years fished, allowing more precise estimates of vessel effects. The choice of top 4 vessels falls between the top 5 used by Vaughan et al. (2001) and the top 3 used by MacCall et al. (2011). We note that these are not exactly the same vessels because the previous authors were analyzing shorter time series.

Generalized Additive Models (GAMs) were fit to the CPD data and a set of potential explanatory variables. We used GAMs to accommodate potentially non-linear responses to variables such as depth (Wood 2017). The Generalized Linear Models used in previous studies can be considered a special case of GAMs with only linear responses. The set of potential predictor variables included fishing year, season, month, a smooth function of depth, vessel ID, and fishing area, coded as a factor. The base model included year only, and other variables were added in a forward selection process. Model selection was based on the Akaike Information Criterion (AIC) and model diagnostics. A Tweedie error distribution was used because it provided better model diagnostics than a more standard log-normal assumption. The final model had the form:

$$\log(\text{CPD}) \sim \text{gam}(\text{year} + \text{vessel} + \text{s}(\text{month}, \text{bs} = \text{"cc"}), \text{data} = \text{top4}, \text{family} = \text{tw})$$

This model explained 45.5% of the residual deviance. The vessel effect was significant, with one vessel having lower than average catch rates and one vessel considerably higher catch rates. Month was fit with a cyclic spline function to ensure the estimated catch rate was the same at the beginning and start of the year (0=12, Fig. 11).

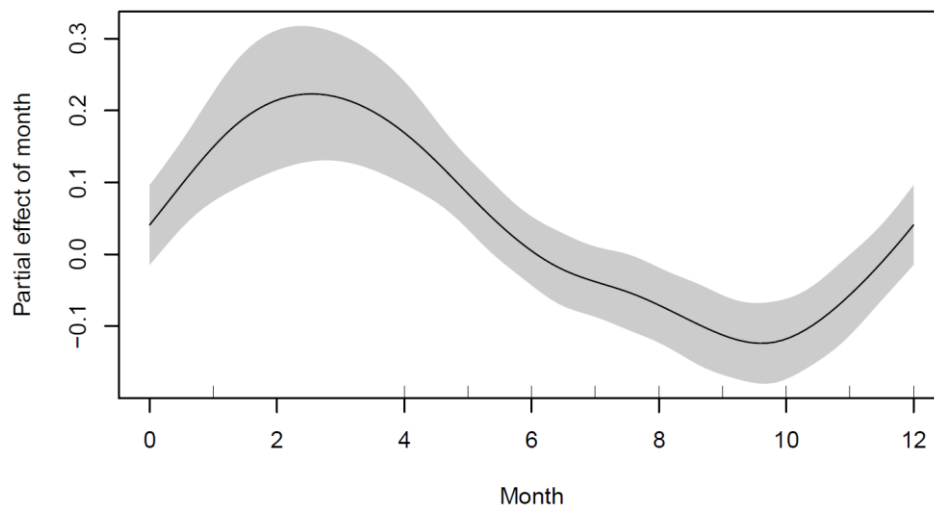


Figure 11. Partial effect of month in the Generalized Additive Model.

According to the estimated seasonal cycle, catch rates would be highest in February and March when the fishery is closed and lowest in October. This pattern makes sense in that the fishery is closed during the spawning season when fish are aggregated.

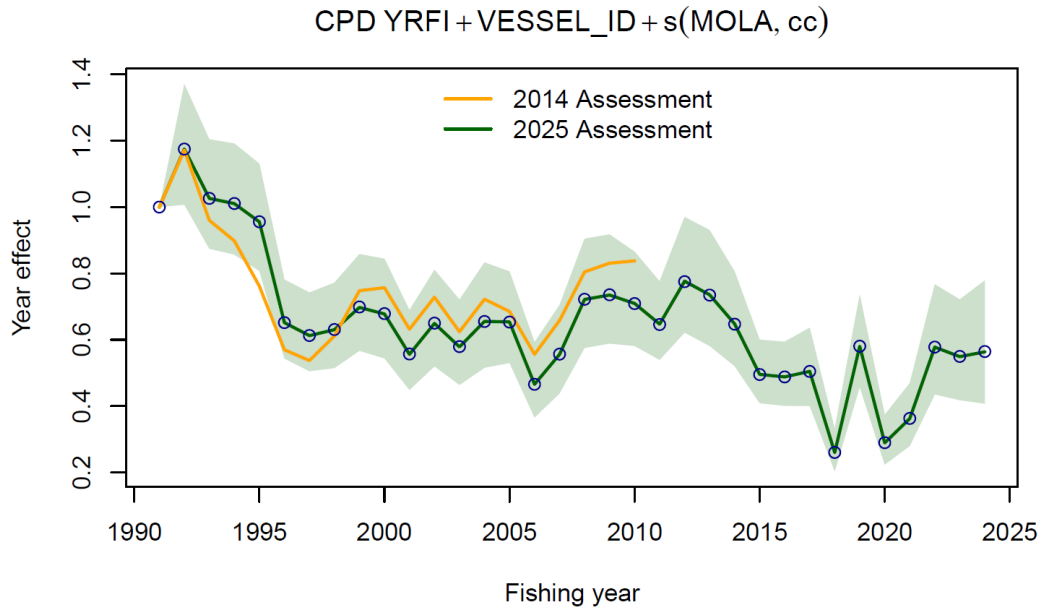


Figure 12. Estimated year effect in the GAM. Solid green line is the estimated effect and the shaded area is ± 2 times the standard error of the estimates. Circles are the estimated year effects estimated with Gamm. For comparison, the orange line is the CPUE index used by Rademeyer & Butterworth (2014) normalized to a value of 1 in 1991.

A final modeling step was to fit vessel as a random effect in a Generalized Additive Mixed-effect Model (Gamm). The mixed-effect model had slightly improved diagnostics but the AIC was higher than the fixed-effect model. The among-vessel standard deviation was 0.31 compared to the within-vessel standard deviation of 3.21. The vessel effects estimated with the mixed-effect model were slightly lower than the corresponding fixed-effect vessel parameters indicating only slight shrinkage. Finally, the year effects estimated with the fixed and mixed-effects models were identical (Fig. 12).

The year variable can be interpreted as a standardized index of relative abundance in the sense that it reflects the catch per day with the effects of vessel and month removed (Fig. 12). Starting from an arbitrary value of 1 in 1991, CPD decreased markedly to 0.65 in 1996 and then declined more gradually to about 0.47 in 2006. Catch rates were higher from 2008 to 2013, declined to lows of about 0.3 in 2018 and 2020 and

rebounded to almost 0.6 in 2022 to 2024. The CPUE index from the 2014 assessment falls within the 95% confidence intervals of the updated index except for 1995.

Size Composition

As an overall measure of size distribution, total numbers landed were divided by landing weight to obtain the mean weight per trip. Mean weight per trip was fairly steady around 32 lbs with low variation between 1991 and 2008 (Fig. 13). There was increased variability in mean weight from 2009 and on, with a pronounced decrease from 2015 to 2021. The team considered several hypotheses to explain the increased variance and lower mean weights during that period:

1. Less precision in reporting numbers than weight landed;
2. Fishing in shallower areas;
3. Fishing in a nursery area;
4. Recruitment of small individuals from outside the main fishing ground.

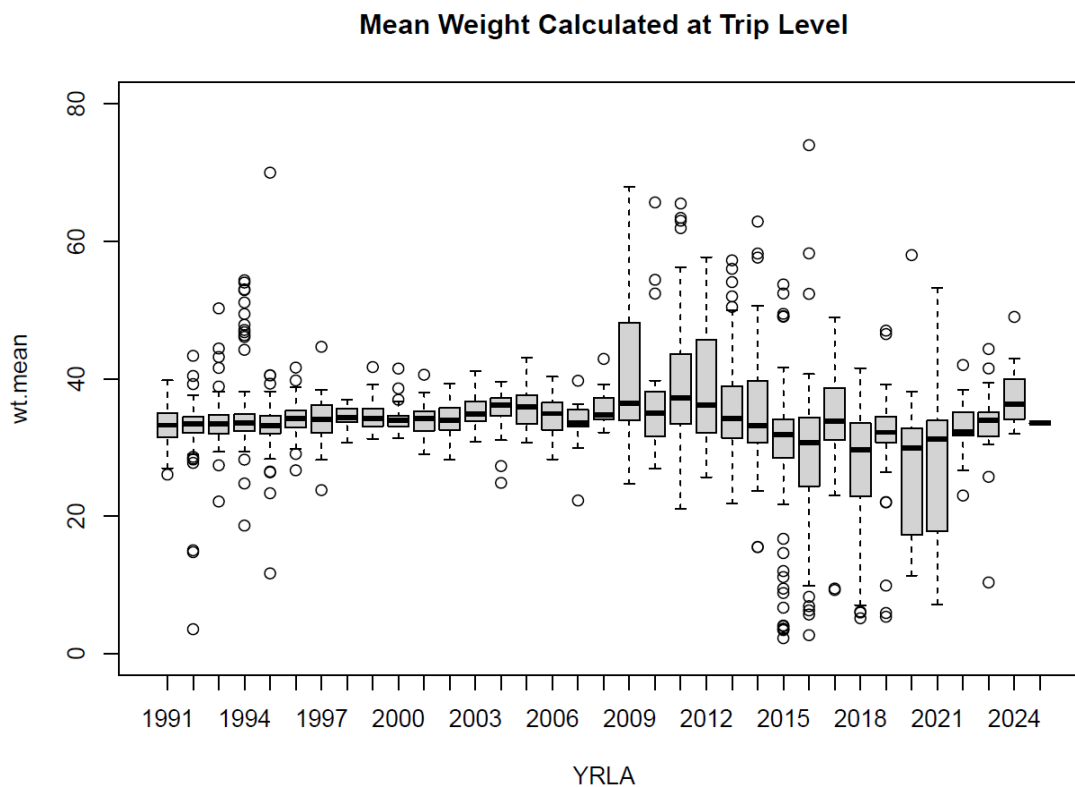


Figure 13. Mean weight (pounds) calculated as reported numbers divided by reported weight. Box plots show the median (black line) interquartile ranges (boxes), range (dashed lines), and outliers (points).

Hypothesis two was not supported because the low mean weights were reported at intermediate depths (Fig. 14). Fishermen at the AIM workshop confirmed that weight is recorded more precisely than number landed because the coupons and payment are based on the landed weight. Given this support for Hypothesis one, the LPUE index was based on weight not numbers and size composition was based on length samples.

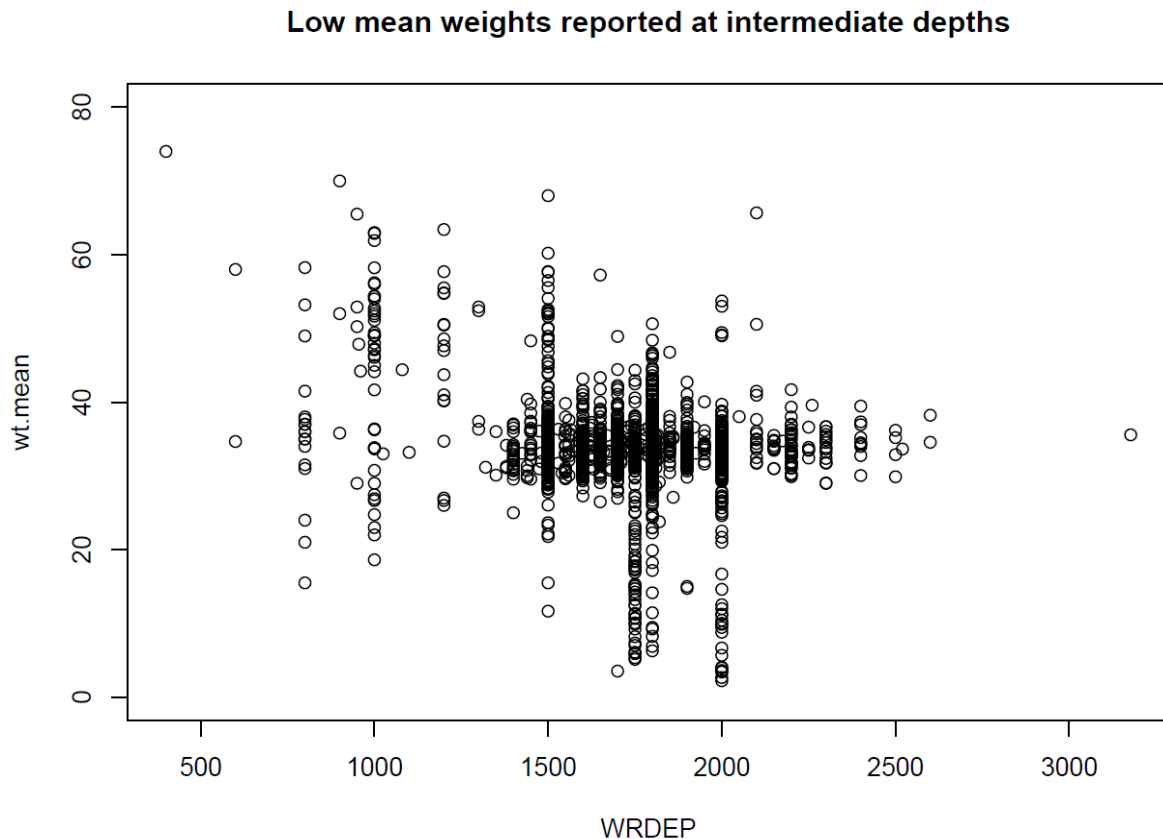


Figure 14. Mean weight (pounds) per trip plotted against fishing depth (ft). Each point represents one fishing trip.

Length data are available from 1984 until 2023. During these years, 20,485 length measurements were reported from 773 trips. For consistency, standard lengths and total lengths were converted to fork lengths with the conversions provided by Buble et al. (2025). Length samples were aggregated by fishing year and the distributions have been weighted by the total landings in each trip containing length samples.

Most of the reported lengths are between 80 and 110 cm (Fig. 15). Smaller fish were reported in 1984 but this was a small sample (n=6) during the very early years of the fishery. The smallest reported fish (14cm in 1991) was not used in the analysis. It was likely a 144 cm fish that was misrecorded. There were also smaller fish (20-40 cm)

reported in 2011 but not in the years 2015 to 2021 (Fig. 15). Larger wreckfish were reported in the earlier years (1988-1996 and 1998-2005), but overall, the length distributions have remained stable over time and consistent with the previous assessment.

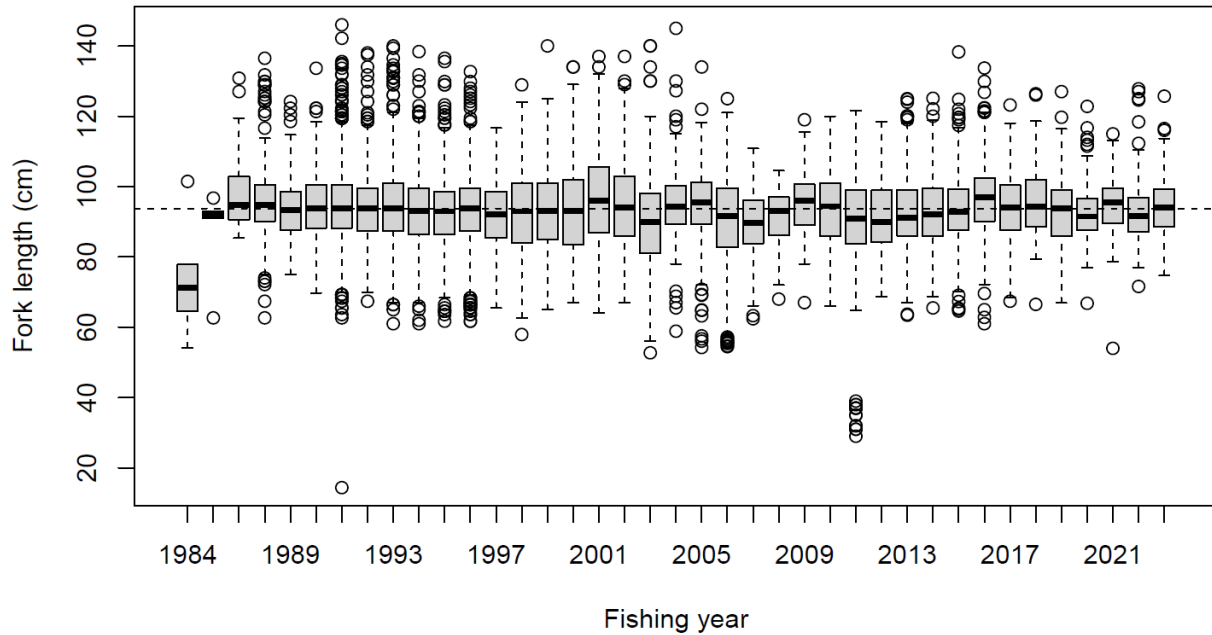


Figure 15. Boxplots of length distributions from 1984 to 2024. Fishing year 1985 is missing from this plot because no length samples were taken that year. The dashed line is the grand mean length 93.7cm.

Life History Data

The life-history information and allometric relationships data needed for the stock assessment are listed in Table 1. The uncertainty in these parameters will be retained for sensitivity analyses.

Table 1. Demographic parameters and relationships obtained from Rademeyer & Butterworth (2014) and Bublely et al. (*in press*).

Relationship/parameter				
Natural mortality rate	M	0.037 (yr ⁻¹)	SD = 0.00407	
Von Bertalanffy growth equation	L _∞ (cm)	K (yr ⁻¹)	t ₀ (yr)	
Female	102.6	0.10	-5.30	
Male	94.9	0.11	-5.40	
Maturity	L _{50%}	76.8 (cm)	L _{90%}	86.0 (cm)
Stock-recruitment relationship	Steepness (h)	0.75		
Length-weight relationship	a	0.0000208	b	3.017

Assessment model

The Stock Assessment Continuum Tool (Cope 2024) has been chosen as the primary assessment method for this project. This tool provides a Shiny App interface to the powerful Stock Synthesis assessment program (Methot & Wetzel 2013). Depending on data availability, the continuum tool can fit models of varying complexities ranging from catch-only models for data-limited stocks to fully age-structured models. This continuum is appropriate for a data-moderate stock such as wreckfish. Application to wreckfish started with a length-only model and sequentially added landings and landings per unit effort (LPUE). Further development will examine the available size-composition data.

Several model variants have been fit:

1. Likelihood profiles and sensitivity analysis of key life-history parameters;
2. Asymptotic or dome-shaped selectivity of the commercial fishery;
3. Assumptions about stock-recruitment steepness and recruitment deviations;

A production model was proposed as a secondary assessment method. The production model is given lower priority because it relies heavily on the LPUE index and doesn't make use of the length distribution data.

Base model

The base model incorporates landings from 1987-2024, LPUE from 1991-2024, and length composition from 1987-2023. The start year, 1987, includes the initial buildup of the fishery (Fig.1) and is the first year with reliable length-composition data (Fig. 15).

Life-history parameters are listed in Table 1 and plotted in Fig.16. The growth parameters indicate a species that grows rapidly to an asymptotic length by about age 30. A 100-cm wreckfish could be anywhere between 30 and 100 years old. The estimated selectivity ogive indicates that full selectivity is reached at approximately 105cm.

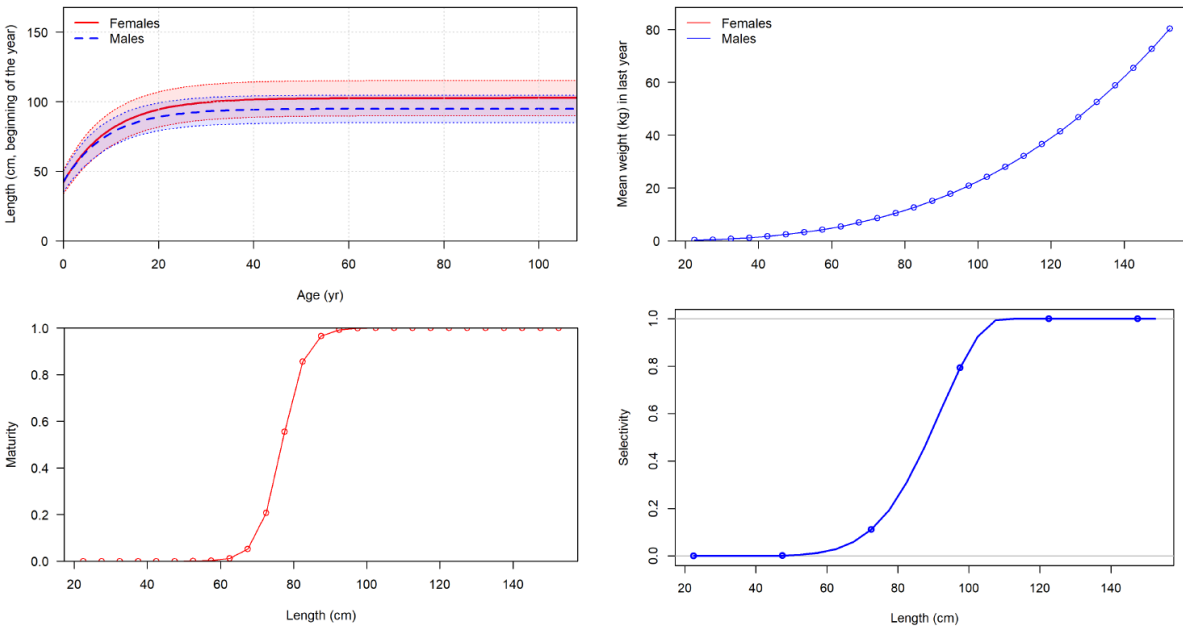


Figure 16. Wreckfish life-history relationships from Bubley et al. (*in press*). The size-selectivity curve is estimated in the base model.

Additional specifications include σ_R , which measures deviations of year-class size (on the log scale) from the Beverton-Holt stock-recruitment curve. The base level $\sigma_R=0.5$ is an appropriate default for a long-lived species such as wreckfish. Bias adjustment was applied to the recruitment deviations according to the methodology of Methot & Taylor (2011).

The base model provides an acceptable fit to the length composition data and LPUE index (Fig. 17). The estimated lengths slightly overestimate the lower tail and underestimate the upper tail. The estimated peak length is correct but extends into the next 5-cm length bin. The estimated LPUE index declines rapidly from 1991-1995 and then more gradually until the end of the time series, capturing the trend but not the peaks and dips.

The estimated spawning biomass started near 4000 t, declined rapidly from 1990 to 1997 and then more gradually to about 1000 t (Figure 18).

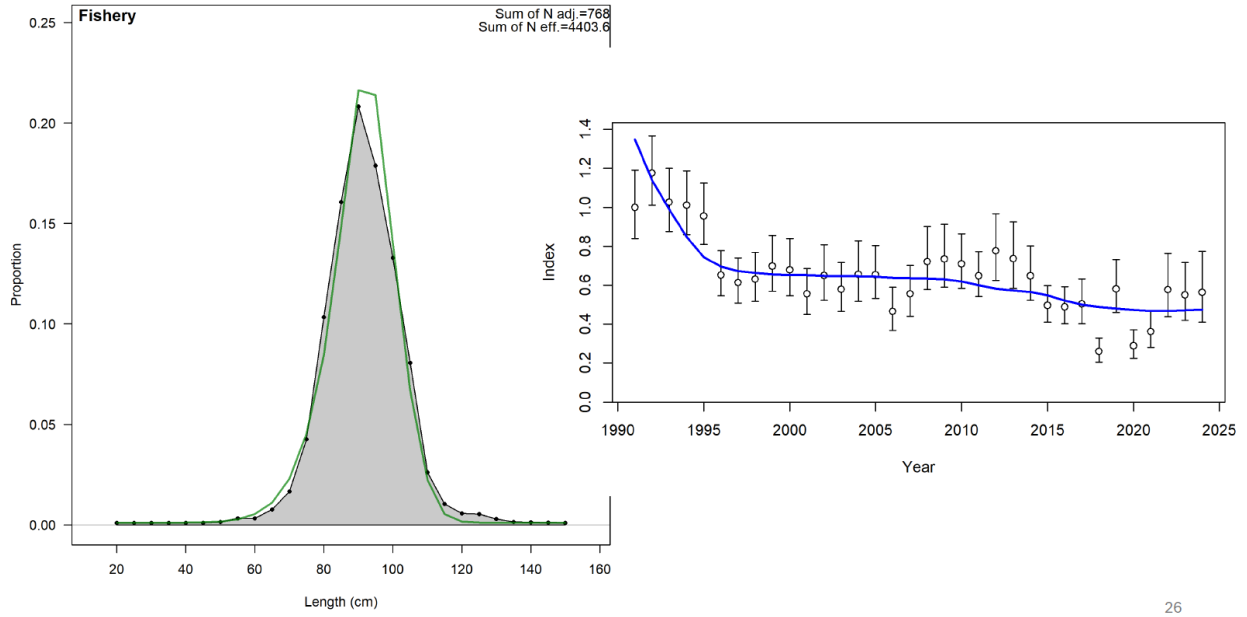


Figure 17. A. Fit to the aggregated length-frequency data. The gray area represents the aggregated data and the green line the fitted distribution. B. Fit to the standardized LPUE index. The blue line is the model estimate.

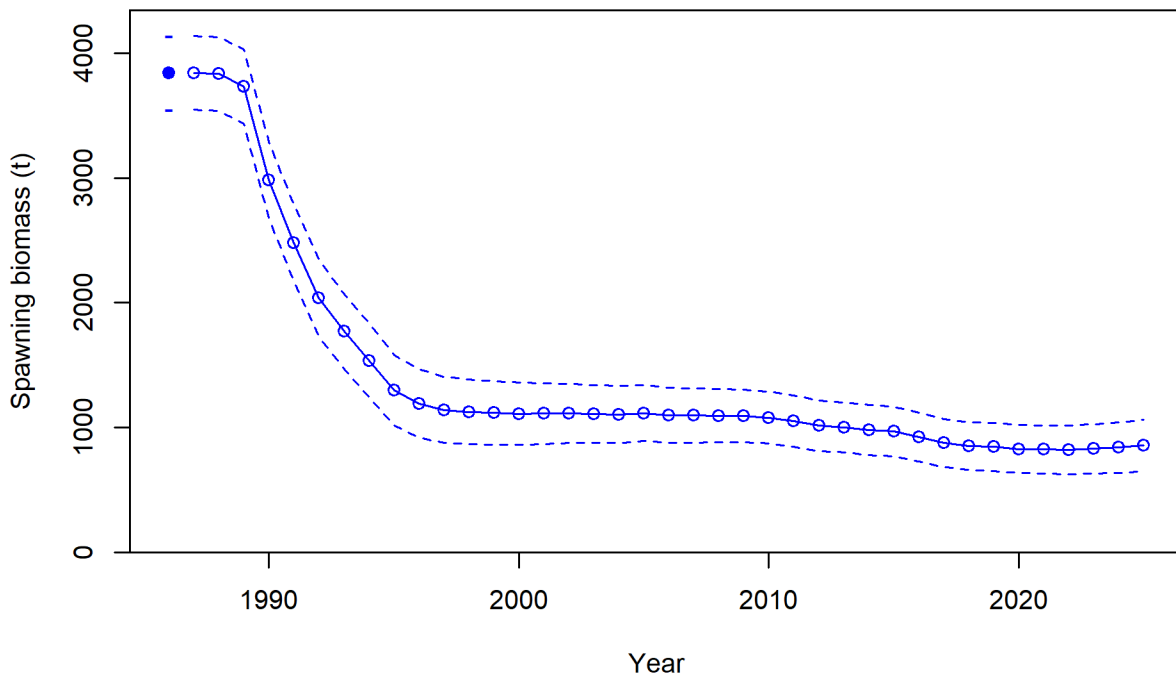


Figure 18. Estimated spawning biomass with 95% confidence intervals.

Fishing mortality (F) was highest from 1990 to 1995 (Fig. 19). There was a gradual increase in F from 2010 to 2016 and decline since then to about 0.05. Mean age in the population declined from over 25 years to about 15 at the end of the timeseries. The equilibrium yield curve peaks at over 100t at a fraction (~ 0.26) of the unfished spawning biomass. The current depletion level (0.22) is to the left of the MSY level.

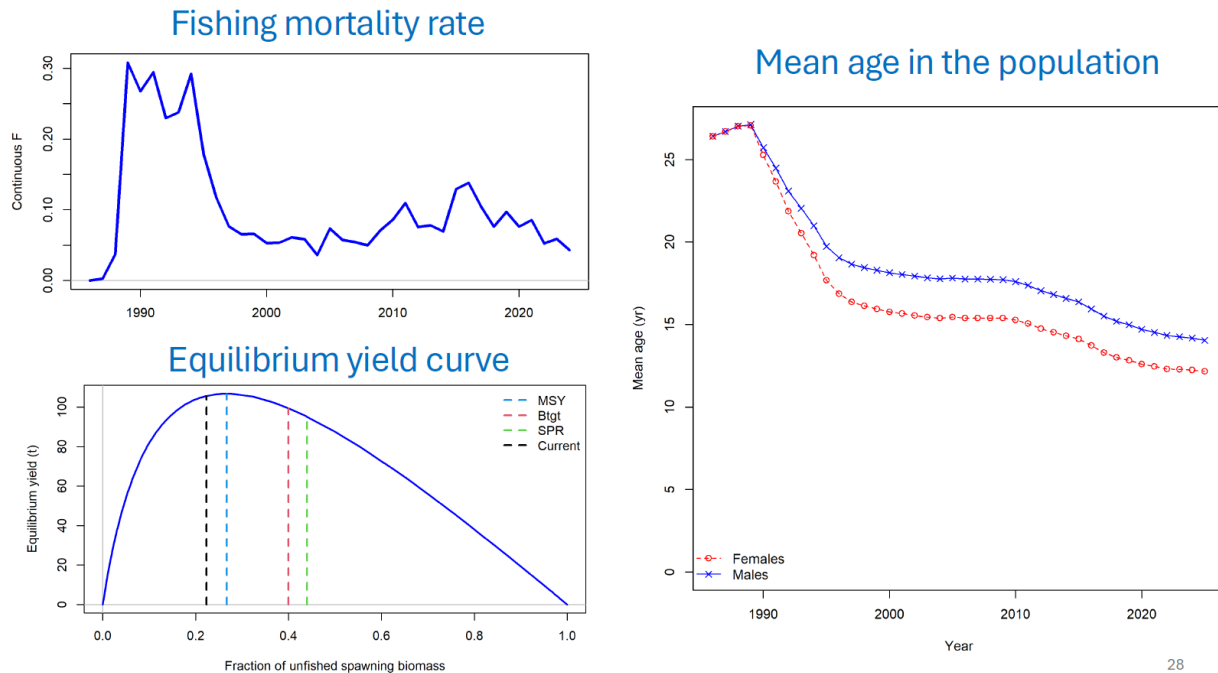


Figure 19. Estimated fishing mortality rate, mean age in the population and equilibrium yield. The biomass target (red line) and SPR level (green line) are default levels that don't pertain specifically to wreckfish.

Sensitivity Analyses and Likelihood Profiles

Three levels of steepness were chosen for sensitivity runs: 0.6, 0.75, and 0.9. The three steepness levels resulted in very similar estimates of Spawning biomass but different recruitment trajectories (Fig. 20). The higher steepness value (0.9), estimated increasing recruitment at the end of the time series, while the lower value (0.6) estimated stable and lower recruitment.

The different steepness values result in different equilibrium yield curves (Fig. 21). Compared with the base value of 0.75, the lower steepness value (0.6) results in a more symmetric yield curve with lower maximum yield. By contrast, the higher steepness results in a more skewed yield curve with higher maximum yield. The likelihood profile for steepness has negative log-likelihood decreasing with steepness, suggesting a better fit with lower steepness (Fig. 21). However, this decrease is driven mainly by the estimated recruitment; the likelihood components associated with the length data and

LPUE index are all within one log-likelihood unit. These fishery-dependent data are uninformative about recruitment.

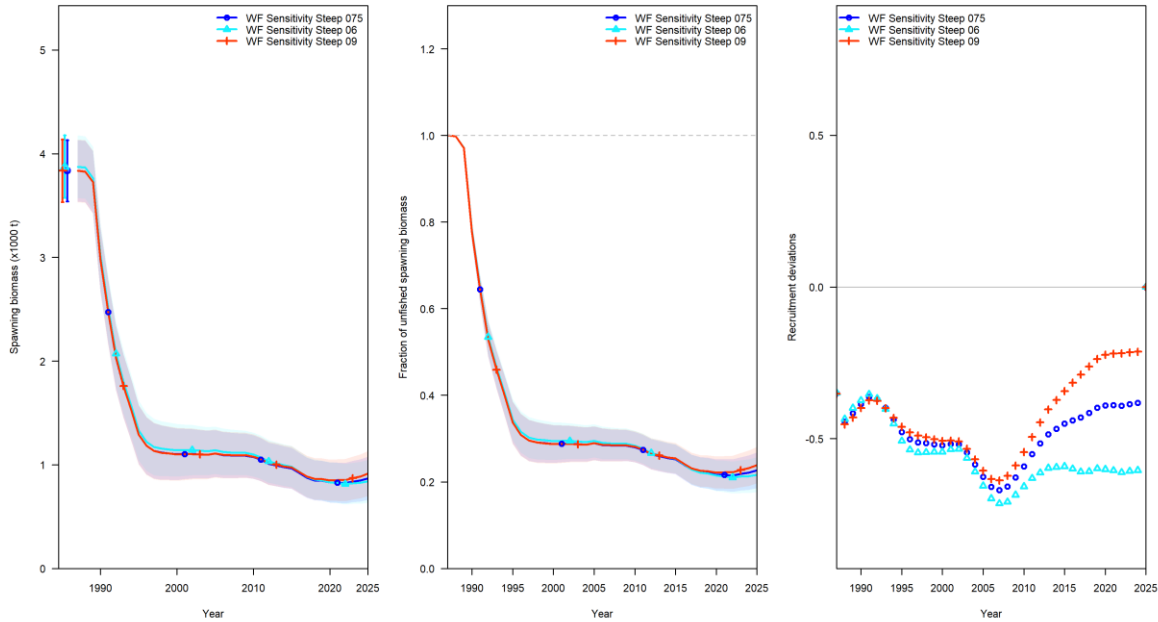


Figure 20. Sensitivity runs for stock-recruitment steepness.

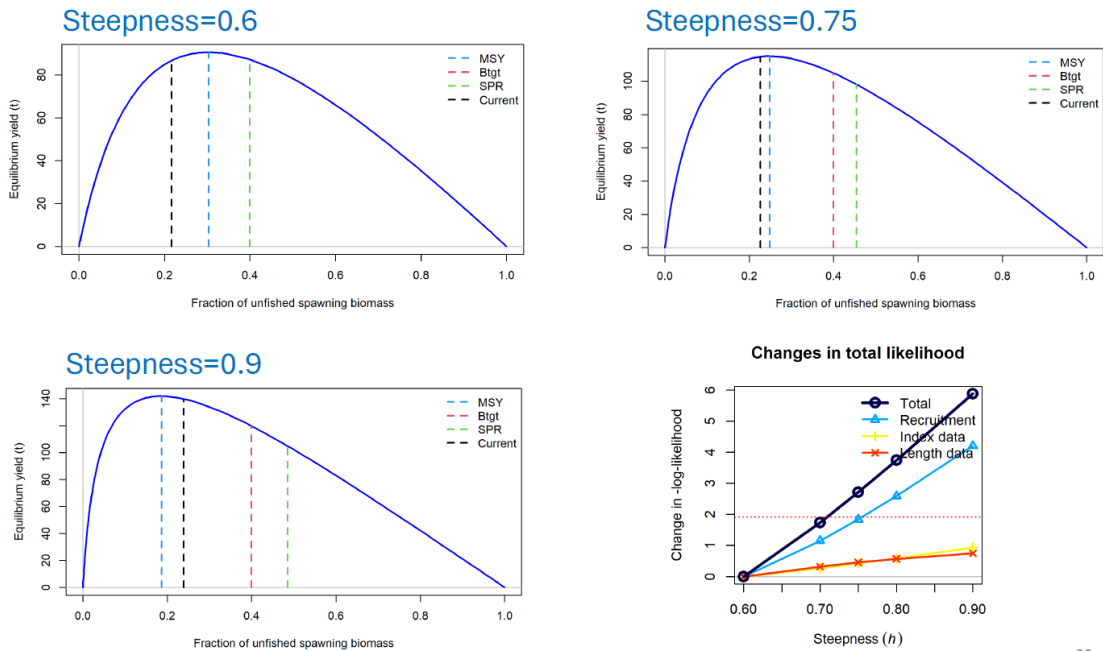


Figure 21. Sensitivity runs and likelihood profiles for stock-recruitment steepness.

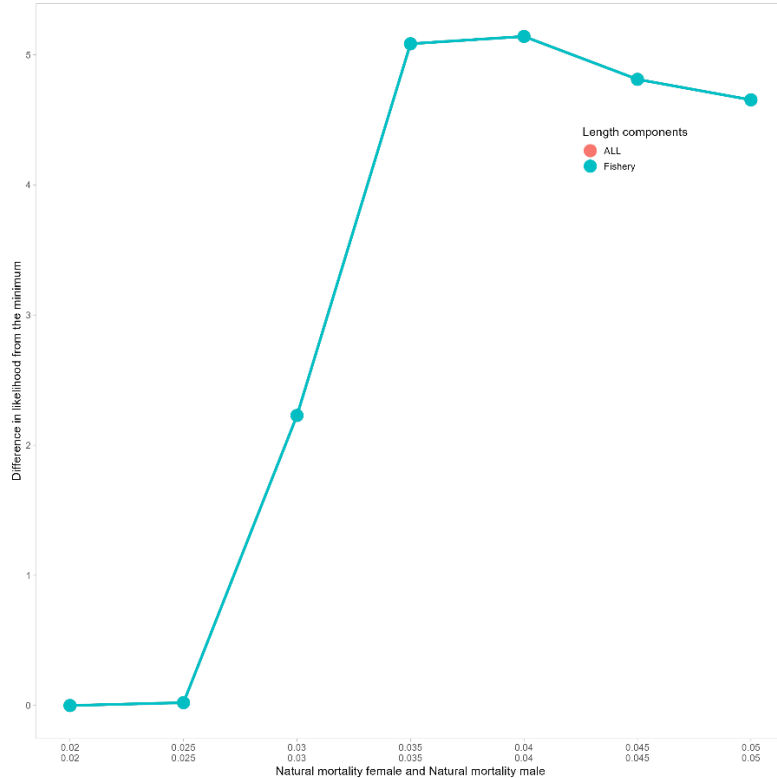


Figure 22. Likelihood profile for the natural mortality rate.

A likelihood profile was calculated for a range of natural mortality rates (M) from 0.02 to 0.05. Among the runs, there was little difference in the estimated spawning biomass trajectories (not shown). The difference in negative log likelihood drops markedly for values of $M < 0.035$ (Fig. 22). This difference is due to the component likelihood for the length composition data. While this profile suggests more support for lower M values, there is no minimum, indicating that M isn't estimable from the available data.

A sensitivity analysis was also conducted for size selectivity to the fishery, comparing the asymptotic selectivity of the base run with a dome-shaped selectivity curve, which might be expected in a hook fishery. Although the initial parameters specified a clearly dome-shaped selectivity curve, the final parameters indicated a selectivity curve that declined only slightly at lengths greater than those typically caught in the fishery. As a result, there were no differences in estimated spawning biomass or recruitment with the dome-shaped model (figures not shown).

A sensitivity analysis was conducted for recruitment variability around the estimated stock-recruitment curve. The parameter σ_R specifies the standard deviation of recruitment deviations on a log scale. The base value of 0.5 is typical for a long-lived demersal species such as wreckfish. Lower σ_R resulted in different estimated trajectories of spawning biomass. Recruitment in the base model decreased to a minimum around 2007 before increasing toward the end of the timeseries (Fig. 23).

However, these recruitment estimates are highly uncertain. Sensitivity runs were therefore made with σ_R reduced to 0.25 and 0. As expected, reducing σ_R results in a smoother trajectory of recruitment estimates with narrower confidence intervals.

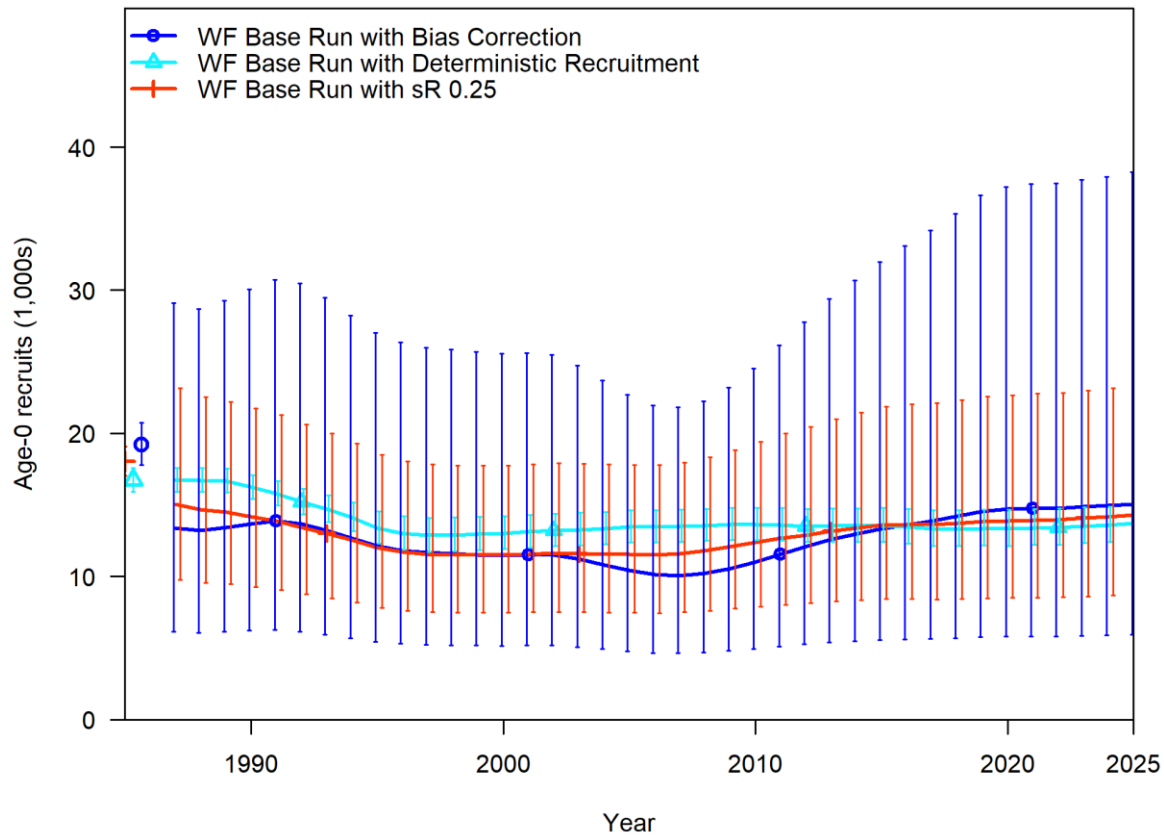


Figure 23. Sensitivity runs with different values of σ_R : 0.5 (dark blue), 0.25 (red) and 0 (light blue).

In the deterministic run ($\sigma_R=0$), recruitment initially declines as spawning biomass was reduced, then levels out as spawning biomass stabilizes. There is no statistical basis for specifying σ_R ; the choice is informed by the species life history and observations of recruitment variability (if they exist). The different values of σ_R affect the estimated selectivity and equilibrium yield curves (Fig. 24). Compared with the base run, the effect of decreasing σ_R is to shift the length at full selectivity to the right. With $\sigma_R=0$, selectivity is reduced to about 0.5 at the asymptotic length (L_{inf}). The equilibrium yield curves become slightly more symmetric with lower maximum yield.

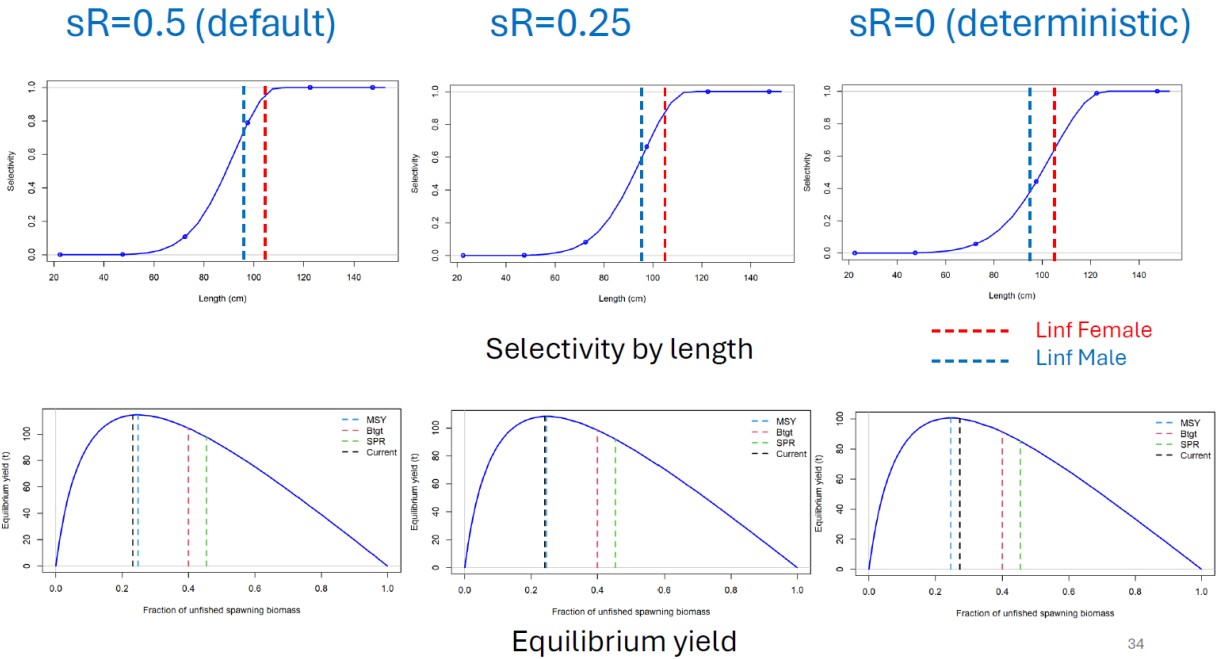


Figure 24. Estimated size selectivity and equilibrium yield curves for sensitivity runs with different values of σR .

Four model variants are retained as candidate operating models for harvest strategy evaluation (Fig. 25). The variants differ in assumptions made about recruitment dynamics, which is a major source of uncertainty in the wreckfish assessment that can't be resolved with the available data. The three models in the top row differ in the steepness of the stock-recruit relationship, which affects the maximum equilibrium yield and the level of stock depletion at which the maximum occurs. The deterministic recruitment model is included because it implies different size selectivity to the fishery. This set of candidate models may be modified based on expert review, further developments in fitting the stock assessment model, or feedback from the harvest strategy evaluation.

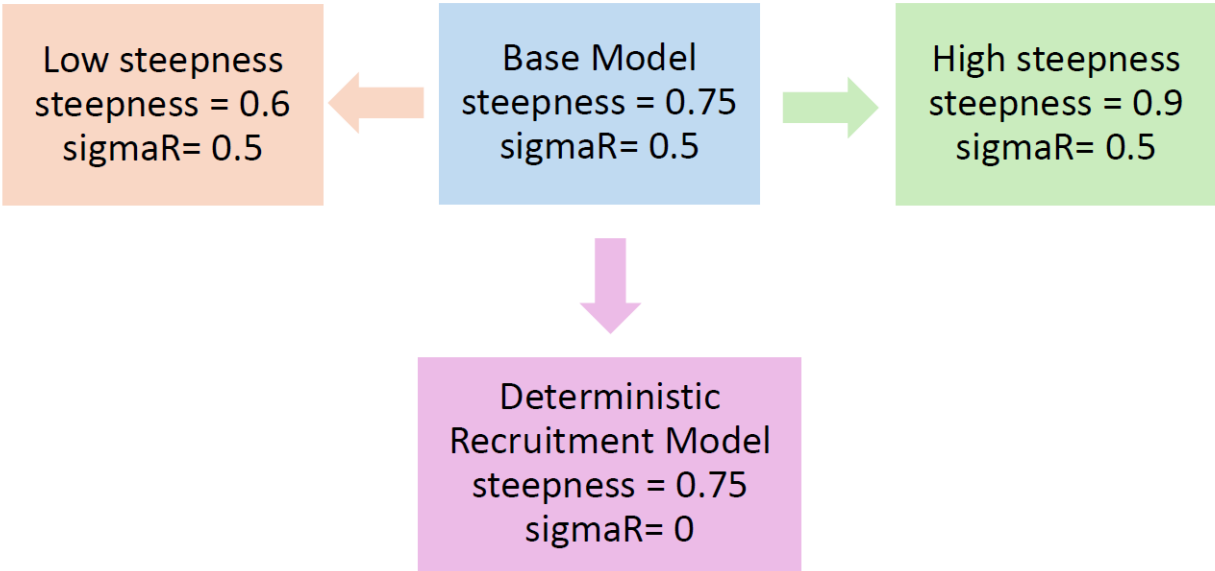


Figure 25. Candidate operating models for harvest strategy evaluation.

Incorporation of Age-composition Data

The South Carolina Department of Natural Resources has performed age determination of a set of otoliths to derive an updated growth curve for wreckfish (Bublely et al. *in press*). The von Bertalanffy parameters are reported in Table 1. The numbers of otoliths collected in port sampling of the wreckfish fishery are listed in Table 2.

Table 2. Numbers of wreckfish otoliths sampled for age analysis by year and state.

Year	FL	SC	Unknown	Total
1991	154		69	223
1992	61			61
1993	18		48	66
1994	57		9	66
1995	1		67	68
1996	141			141
1997	54		1	55
1998	14			14
1999	19			19

2004	120			120
2005	220	7		227
2006	115	23		138
2007	37	8		45
2008	8	18		26
2009	8	24		32
2010	64	33		97
2011	24	132		156
2012		34		34
GrandTotal	1115	279	194	1588

These otoliths fall into two groups: from 1991 to 1999 most of the otoliths were sampled in Florida, whereas from 2004 to 2012 samples were collected in FL and SC.

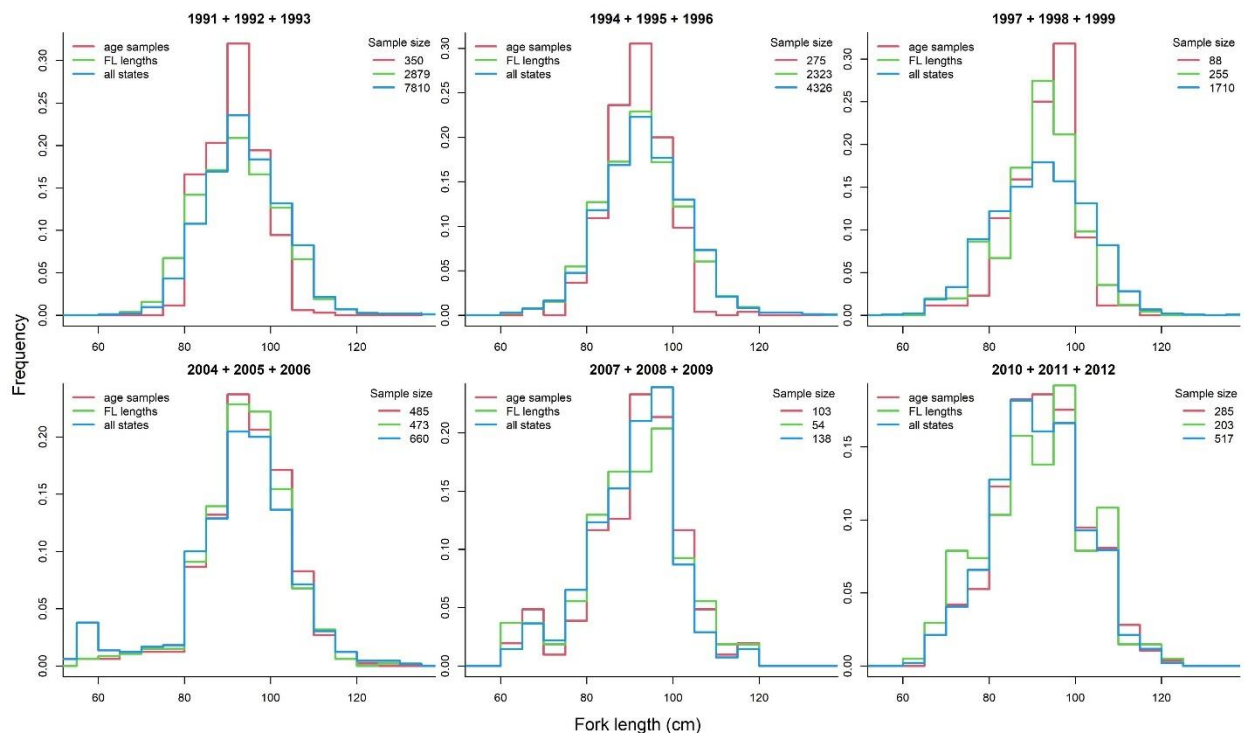


Figure 26. Length distributions of wreckfish from portside sampling. The length data are aggregated in three-year time blocks.

As a first step, we compared the length distributions of fish from which otoliths were sampled to the distribution of measured lengths in Florida and all states (Fig. 26). In the early time period, the modes of the distributions agree but the age samples under-sampled the tails of the distributions. In the latter period there is much better agreement among the length distributions. This comparison suggests that the age composition data may represent the wreckfish fishery, at least in the latter time period.

We propose to incorporate the age-composition data into the stock assessment in three steps:

1. Enter age-composition as marginal data, which means they don't contribute to the likelihood (goodness of fit). However, this allows comparison of the observed and estimated age compositions.
2. Determine whether the age compositions are representative of the wreckfish population.
3. If yes, enter age-composition as conditional on the length distribution, in which case they would be included in the model likelihood.

This analysis will be conducted in the next phase of the stock assessment.

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